Higgs-Field and a New Scalar-Tensor Theory of Gravity

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Abstract

The combination of Brans and Dicke's idea of a variable gravitational constant with the Higgs-field mechanism results in a renormalizable theory of gravity. Einstein's theory is realized after symmetry breaking in the neighbourhood of the Higgs-field ground-state.

There exist today in the literature two fundamental scalar fields connected with the mass problem. First of all Brans and Dicke [1] introduced a scalar field with the intention following Mach's principle [2], that the active as well as passive gravitational mass $m_0\sqrt{G}$, that means the gravitational "constant" G, is not a constant but a function determined by the other particles of the Universe. In this way also the problem of Dirac's large cosmological numbers should be solved. Secondly, in elementary particle physics the inertial mass m_0 is generated with respect to the gauge invariance by the interaction with the scalar Higgs-field, the source of which is also given by the particles in the Universe [3]. Because of the identity of gravitational and inertial mass (equivalence principle) it seems to be meaningful, if not even necessary to identify these two approaches. Then the Lagrange-density has the unique form ($\hbar = 1,\ c = 1$):

$$\mathcal{L} = \left[\frac{1}{16\pi}\alpha\phi^\dagger\phi R + \frac{1}{2}\phi^\dagger_{||\mu}\phi^{||\mu} - V(\phi)\right]\sqrt{-g} + L_M\sqrt{-g} \tag{1}$$

with the Higgs-potential (μ^2, λ real valued constants)

$$V(\phi) = \frac{\mu^2}{2}\phi^\dagger\phi + \frac{\lambda}{4!}(\phi^\dagger\phi)^2 + \frac{3}{2}\frac{\mu^4}{\lambda}\,. \tag{1a}$$

Herein ϕ is an U(N) iso-vector, $||\mu$ means its covariant derivative, R is the Ricci-scalar and α a dimensionless factor, whereas L_M contains the fermionic and massless bosonic fields belonging to the inner gauge-group $U(N)$. Obviously, the positive Higgs-field quantity $\phi^\dagger\phi$ (c.f. eq. (9a)) plays the role of a variable reciprocal gravitational "constant". Formula (1) is related to a generalization of Brans and Dicke's theory proposed by Bergmann [4] and Wagoner [5] as well as by Zee [6]. We want to point here to some interesting features of the ansatz (1), which unifies gravity with the other interactions using a minimum of effort.

Before symmetry breaking the theory following from (1) contains no gravitational constant; the only dimensional free parameters are those of the

Higgs-potential. Such a theory of gravity may be renormalizable according to the criterion given by de Witt [7], although it is not unitary. - Concerning symmetry breaking the ground-state of the Higgs-field is given by ($\mu^2 < 0$)

$$\phi_0^\dagger \phi_0 = v^2 = \frac{-6\mu^2}{\lambda} \tag{2}$$

with

$$V(\phi_0) = 0. \tag{2a}$$

By this ground state the quantity

$$G = (\alpha v^2)^{-1} \tag{3}$$

related to Newton's gravitational constant (see below), as well as the mass of the gauge bosons

$$M_W = \sqrt{\pi}\, g v \tag{4}$$

are determined (g coupling constant of the gauge group $U(N)$). Accordingly the factor α means the ratio

$$\alpha \simeq (M_P/M_W)^2 >> 1, \tag{5}$$

where M_P is the Planck mass.

The field equations for gravity and Higgs-field following from (1) take the form:

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \frac{8\pi}{\alpha\phi^\dagger\phi} V(\phi) g_{\mu\nu} = \\ = -\frac{8\pi}{\alpha\phi^\dagger\phi} T_{\mu\nu} - \frac{4\pi}{\alpha\phi^\dagger\phi} \left[\phi^\dagger_{||\mu}\phi_{||\nu} + \phi^\dagger_{||\nu}\phi_{||\mu}\right] + \\ + \frac{4\pi}{\alpha\phi^\dagger\phi} \phi^\dagger_{||\lambda}\phi^{||\lambda} g_{\mu\nu} - \frac{1}{\phi^\dagger\phi}\left[(\phi^\dagger\phi)_{||\mu||\nu} - (\phi^\dagger\phi)^{||\beta}{}_{||\beta}\, g_{\mu\nu}\right] \end{aligned} \tag{6}$$

and

$$\phi^{||\mu}{}_{||\mu} - \frac{1}{8\pi}\alpha\phi R + \mu^2\phi + \frac{\lambda}{6}(\phi^\dagger\phi)\phi = 0 \tag{7}$$

as well as the adjoint equation of (7). Herein $T_{\mu\nu}$ is the symmetric energy momentum tensor belonging to $L_M\sqrt{-g}$ in (1) alone. The conservation laws of energy and momentum read

$$T_{\mu}{}^{\nu}{}_{||\nu} = 0\,. \tag{8}$$

Now we perform the symmetry breaking and introduce the unitary gauge. If with respect to (2)

$$\phi_0 = vN; \quad N^\dagger N = 1; \quad N = \text{const.} \tag{9}$$

represents the ground-state, the Higgs-field ϕ can be brought within the unitary gauge into the form:

$$\phi = \rho N, \quad \rho^2 = \phi^\dagger\phi. \tag{9a}$$

For the following we use therefore instead of ϕ the real valued field quantity

$$\varphi = \rho/v \tag{10}$$

($\varphi = 1$ represents the ground-state). Restricting ourselves to the field equations for gravity i.e. the metric $g_{\mu\nu}$ and the Higgs-field φ we find from (6) and (7) ($|\mu$ means the usual partial derivative):

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \frac{12\pi}{\alpha v^2}\frac{\mu^4}{\lambda}\varphi^{-2}(\varphi^2-1)^2 g_{\mu\nu} = \\ = -\frac{8\pi}{\alpha v^2}\varphi^{-2}T_{\mu\nu} - \frac{8\pi}{\alpha}(1+\frac{\alpha}{4\pi})\varphi^{-2}\varphi_{|\mu}\varphi_{|\nu} + \\ +\frac{4\pi}{\alpha}(1+\frac{\alpha}{2\pi})\varphi^{-2}\varphi^{|\lambda}\varphi_{|\lambda}g_{\mu\nu} - \\ -2\varphi^{-1}\left[\varphi_{|\mu||\nu} - \varphi^{|\lambda}{}_{||\lambda}\,g_{\mu\nu}\right] \end{aligned} \tag{11}$$

and

$$\frac{4\pi}{\alpha}(1+\frac{3\alpha}{4\pi})\varphi^{2|\mu}{}_{||\mu} + \frac{48\pi}{\alpha v^2}\frac{\mu^4}{\lambda}(\varphi^2-1) = \frac{8\pi}{\alpha v^2}T\,. \tag{12}$$

With respect to (3) and (5) we obtain from (11) and (12) the final result:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + 12\pi G \frac{\mu^4}{\lambda} \varphi^{-2} (\varphi^2 - 1)^2 g_{\mu\nu} =$$

$$= -8\pi G \varphi^{-2} T_{\mu\nu} - 2\varphi^{-2} \varphi_{|\mu} \varphi_{|\nu} +$$

$$+2\varphi^{-2} \varphi^{|\lambda} \varphi_{|\lambda}\, g_{\mu\nu} - 2\varphi^{-1} \left[\varphi_{|\mu||\nu} - \varphi^{|\lambda}{}_{||\lambda} g_{\mu\nu} \right] \tag{13}$$

and

$$\varphi^{2|\mu}{}_{||\mu} + 16\pi G \frac{\mu^4}{\lambda} (\varphi^2 - 1) = \frac{8\pi G}{3} T \,. \tag{14}$$

There are two very important differences with respect to Brans and Dicke's scalar tensor theory. First, the scalar field φ possesses a finite range $l = M_\varphi^{-1}$ corresponding to the mass term in (14) according to which the excited Higgs-field has the mass square:

$$M_\varphi^2 = 16\pi G \frac{\mu^4}{\lambda} \,. \tag{15}$$

This is smaller than the usual value by the factor α^{-1}. In this connection we note, that G in (3), (13) and (14) represents Newton's gravitational constant only up to a factor of the order of one. The exact connection between G and the Newtonian value G_N is given by the Newtonian limit of (13) and (14) and this depends, as shown below, on the value of l for the range of the scalar field. In case of a suitable choice of this range also no difficulties with respect to the solar-relativistic effects or gravitational waves are to be expected; however the possibility of a fifth force of Yukawa type is given.

Secondly there exists according to the left hand side of (13) a cosmological function (instead of a cosmological constant)

$$\lambda(\varphi) = 12\pi G \frac{\mu^4}{\lambda} \varphi^{-2} (\varphi^2 - 1)^2, \tag{16}$$

which is necessarily positive. This is very interesting because a positive value of a cosmological function (constant) corresponds to a positive mass density, so that this theory could solve the problem of missing mass in cosmology automatically.

For the ground-state of the Higgs-field ($\varphi \equiv 1$) the cosmological function $\lambda(\varphi)$ vanishes (see also (2a)) and from (13) and (14) it follows:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -8\pi G T_{\mu\nu}, \quad T = 0. \tag{17}$$

This is Einstein's theory with light-like matter. Einstein's theory is realized only after symmetry breaking in the neighbourhood of the ground-state. Of course, in case of vanishing energy momentum tensor the ground-state is realized by the Minkowski space-time and $\varphi = 1$.

Finally we investigate the Newtonian limit. For this we set

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}; \quad \varphi = 1 + \zeta \tag{18}$$

and linearize with respect to $|h_{\mu\nu}| << 1$ and $|\zeta| << 1$ ($\eta_{\mu\nu} =$ diag$(1, -1, -1, -1)$. In this way we obtain from (13) and (14) using the de Donder gauge $h_\mu{}^\nu{}_{|\nu} - \frac{1}{2} h_{|\mu} = 0$:

$$h_{\mu\nu}{}^{|\lambda}{}_{|\lambda} = -16\pi G (T_{\mu\nu} - \frac{1}{3} T \eta_{\mu\nu}) +$$

$$+32\pi G \frac{\mu^4}{\lambda} \zeta \eta_{\mu\nu} - 4\zeta_{|\mu|\nu} \tag{19}$$

and

$$\zeta^{|\lambda}{}_{|\lambda} + 16\pi G \frac{\mu^4}{\lambda} \zeta = \frac{4\pi G}{3} T\,. \tag{20}$$

Because of the geodesic equation of motion of a free point particle in consequence of (8)

$$h_{00} = 2\phi_N \tag{21}$$

is valid, where ϕ_N is the Newtonian gravitational potential. Insertion of (21) into (19) yields:

$$\phi_N{}^{|\lambda}{}_{|\lambda} = -8\pi G (T_{00} - \frac{1}{3} T) +$$

$$+16\pi G \frac{\mu^4}{\lambda} \zeta - 2\zeta_{|0|0}\,. \tag{22}$$

For a point particle of mass M at rest in the origin the solution of (20) reads:

$$\zeta = \frac{MG}{3r} e^{-r/l}\,, \quad l^2 = \frac{\lambda}{16\pi G\mu^4}\,. \tag{23}$$

Herewith the solution of (22) for a point particle takes the form:

$$\phi_N = -\frac{MG}{r}\left(1 + \frac{1}{3}e^{-r/l}\right). \tag{24}$$

Consequently $G = G_\infty$ is valid, where G_∞ is the Newtonian gravitational constant G_N determined by a torsion-balance experiment in the laboratory in the case $r >> l$. In case of $r << l$ one finds $G = \frac{3}{4}G_0$ with $G_0 = G_N$. It is interesting that such gravitational potentials, where the usual r^{-1}-potential is supplemented by a Yukawa term, are discussed in connection with the fifth force [8] and in view of the flat rotation curves of spiral galaxies [9].

In the static linear Newtonian limit, the potential equations following from (20) and (22) are

$$\Delta\zeta - \frac{1}{l^2}\zeta = -\frac{4\pi G}{3}\rho\,,$$

$$\Delta\phi_N + \frac{1}{l^2}\zeta = \frac{16\pi G}{3}\rho \tag{25}$$

instead of the Poisson-equation. Herein $\varphi^{-2} = 1-2\zeta$ represents the variability of the gravitational "constant" in first order (cf. eq. (13)); it decreases in view of (23) with decreasing distance from a mass. The cosmological function (16) is of second order in ζ and therefore not yet contained in (25); however, its absolute value increases with decreasing distance from a mass. Finally we note, that the scalar-field ζ acts in the potential equation (25) for ϕ_N as a negative mass-density (anti-gravity), c.f. [9]. The applications of these ideas to modern astrophysical and cosmological questions are in preparation.

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