Higgs-Field Gravity within the Standard Model

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Summary

Within the frame-work of the Glashow-Salam-Weinberg model it is shown that the Higgs-field mediates an attractive scalar gravitational interaction of Yukawa-type between the elementary particles which become massive by the ground- state of the Higgs-field after symmetry breaking.

1. Introduction.

Until now the origin of the mass of the elementary particles is unclear. Usually mass is introduced by the interaction with the Higgs-field; however in this way the mass is not explained, but only reduced to the parameters of the Higgs-potential, whereby the physical meaning of the Higgs-field and its potential remains non-understood.

On the other hand there exists an old idea of Einstein, the so called "principle of relativity of inertia" according to which mass should be produced by the interaction with the gravitational field [1]. Einstein argued that the inertial mass is only a measure for the resistance of a particle against the relative acceleration with respect to other particles; therefore, within a consequent theory of relativity, the mass of a particle should be originated by interaction with all other particles of the Universe, whereby this interaction should be the gravitational one which couples to all particles, i.e. to their masses or energies. He postulated even that the value of the mass of a particle should go to zero, if one puts the particle in an infinite distance of all the other ones.

This fascinating idea was not very successful within Einstein's theory of gravity, i.e. general relativity, although it has caused, that Einstein introduced the cosmological constant in order to construct a cosmological model with finite space, and that Brans and Dicke developed their scalar-tensor-theory [2]. But an explanation of the mass does not follow from it until now.

In this paper we will show, that the successful Higgs-field mechanism lies in the direction of Einstein's idea of producing mass by gravitational interaction; we find, that the Higgs-field as source of the inertial mass has to do something with gravity [3], i.e. it mediates a scalar gravitational interaction between the massive particles, however of Yukawa type. This results from the fact, that the Higgs-field itself becomes massive after symmetry breaking. On the other hand, an estimation of the coupling constants shows that it may be unprobable that this Higgs-field gravity can be identified with any experimental evidence. Perhaps its applicability lies beyond the scope of the present experimental experiences.

2. Gravitational Action of the Higgs-Field.

In a previous publication [3] we have shown approximatively the gravitational interaction of the Higgs-field between massive fermions. In the present paper we extend our investigation in an exact manner on fermions and bosons. Due to this reason we perform our calculations within the well established Glashow-Salam-Weinberg model of electro-weak interaction based on the localized group $SU(2) \times U(1)$, taking into account all families of elementary particles. For this we start with the following definitions: [1]

$$\psi^i = \psi^{m_i} = \begin{pmatrix} \psi^{l_i} \\ \psi^{q_i} \end{pmatrix}, \quad m = l, q \tag{2.1}$$

represents the spinorial wave-functions of the i-th familiy ($i = 1, ..., N_f$), wherein

$$\psi^{l_i} = \psi^{l_i}_L + \psi^{l_i}_R \tag{2.2}$$

is the leptonic part with

$$\psi^{l_i}_L = \begin{pmatrix} \nu^i_L \\ e^i_L \end{pmatrix}, \quad \psi^{l_i}_R = e^i_R, \tag{2.2a}$$

and

$$\psi^{q_i} = \psi^{q_i}_L + \psi^{q_i}_R \tag{2.3}$$

means the part of the quarks with

$$\psi^{q_i}_L = \begin{pmatrix} u^i_L \\ d_L^{\,\prime i} \end{pmatrix}, \qquad \psi^{q_i}_R = \begin{pmatrix} u^i_R \\ d_R^{\,\prime i} \end{pmatrix} \tag{2.3a}$$

and

$$d^{\,\prime i} = U^i_{(c)j} d^j \tag{2.3b}$$

[1] Spinor- and isospin- indices are suppressed.

as the Cabibbo transformed quark wave-functions. Here the left-handed fermions $\psi_L^{l_i}$ and $\psi_L^{q_i}$ are doublets with respect to the localized group $SU(2)$, whereas the right-handed ones $\psi_R^{l_i}$ and $\psi_R^{q_i}$ are singlets. Correspondingly the covariant derivatives take the form:

$$D_\lambda \psi_L^{l_i} = (\partial_\lambda + ig_2 W_\lambda^a \tau_a - \frac{1}{2} ig_1 B_\lambda)\psi_L^{l_i},$$

$$D_\lambda \psi_L^{q_i} = (\partial_\lambda + ig_2 W_\lambda^a \tau_a + \frac{1}{6} ig_1 B_\lambda)\psi_L^{q_i},$$

$$D_\lambda \psi_R^{l_i} = (\partial_\lambda - ig_1 B_\lambda)\psi_R^{l_i},$$

$$D_\lambda \psi_R^{q_i} = D_\lambda \begin{pmatrix} u_R^i \\ d_R^{\prime i} \end{pmatrix} = \begin{cases} (\partial_\lambda + \frac{2}{3} ig_1 B_\lambda) u_R^i, \\ (\partial_\lambda - \frac{1}{3} ig_1 B_\lambda) d_R^{\prime i}. \end{cases} \tag{2.4}$$

Herein τ^a are the generators of the group $SU(2), {W_\lambda}^a$ represent the corresponding gauge-potentials and B_λ is the $U(1)$ gauge-potential with g_1 and g_2 as gauge-coupling constants. The covariant gauge-field strengths are given by the commutators

$$\mathcal{F}_{(2)\mu\nu} = F_{(2)\mu\nu}^a \tau_a = \frac{1}{ig_2}\left[D_\mu^{(2)}, D_\nu^{(2)}\right],$$

$$\mathcal{F}_{(1)\mu\nu} = F_{(1)\mu\nu} Y = \frac{1}{ig_1}\left[D_\mu^{(1)}, D_\nu^{(1)}\right] \tag{2.5}$$

((1) and (2) refer to the group $U(1)$ and$SU(2)$ respectively). Here Y is the $U(1)$-generator of the weak hypercharge in the different representations according to (2.4), where we follow the notation of ref. [5] and not of [4]. Finally we introduce a scalar Higgs-field ϕ belonging to the fundamental representation of $SU(2)$; its covariant derivative reads

$$D_\lambda \phi = (\partial_\lambda + ig_2 {W_\lambda}^a \tau_a + \frac{1}{2} ig_1 B_\lambda)\phi. \tag{2.6}$$

Herewith we construct the gauge invariant minimally coupled Lagrange-density:

$$L = L(\psi) + L(F) + L(\phi), \tag{2.7}$$

where

$$(2.7a) \qquad L(\psi) = i\frac{\hbar}{2}\left[\overline{\psi}_{Lm_i}\gamma^\lambda D_\lambda \psi_L^{m_i} + \overline{\psi}_{Rm_i}\gamma^\lambda D_\lambda \psi_R^{m_i}\right] + h.c.,$$

$$(2.7b) \qquad L(F) = -\frac{\hbar}{16\pi}(F_{(2)\lambda\mu}^{\ a} F_{(2)a}^{\ \lambda\mu} + F_{(1)\lambda\mu} F_{(1)}^{\ \lambda\mu})$$

and

$$(2.7c) \quad L(\phi) = \frac{1}{2}(D_\lambda\phi)^\dagger D^\lambda\phi - \frac{\mu^2}{2}\phi^\dagger\phi - \frac{\lambda}{4!}(\phi^\dagger\phi)^2 - k\phi^\dagger\overline{\psi}_{Rm_i}\hat{x}^{m_i}{}_{n_j}\psi_L{}^{n_j} + h.c..$$

(μ^2, λ, k real parameters of the Higgs-potential and $\hat{x}^{m_i}{}_{n_j}$ Yukawa coupling matrix). The field equations following from Hamilton's action principle result in the wave equations for the left and right handed fermions

$$(2.8a) \qquad i\gamma^\mu D_\mu \psi_L{}^{m_i} - \frac{k}{\hbar}\hat{x}^{\dagger m_i}{}_{n_j}\phi\psi_R{}^{n_j} = 0,$$

$$(2.8b) \qquad i\gamma^\mu D_\mu \psi_R{}^{m_i} - \frac{k}{\hbar}\phi^\dagger\hat{x}^{m_i}{}_{n_j}\psi_L{}^{n_j} = 0,$$

in the Yang-Mills equations

$$(2.9a) \qquad D_\mu F_{(2)a}^{\ \mu\lambda} \equiv \partial_\mu F_{(2)a}^{\ \mu\lambda} - g_2\epsilon_{abc}W_\mu^b F_{(2)}^{c\mu\lambda} = 4\pi j_{(2)a}^{\ \lambda},$$

$$(2.9b) \qquad \partial_\mu F_{(1)}^{\ \mu\lambda} = 4\pi j_{(1)}^{\ \lambda}$$

(ϵ_{abc} Levi-Civita symbol) with the current densities

$$(2.10a) \qquad j_{(2)a}^{\ \lambda} = g_2\overline{\psi}_{Lm_i}\gamma^\lambda\tau_a\psi_L{}^{m_i} + i\frac{g_2}{2\hbar}\left[\phi^\dagger\tau_a D^\lambda\phi - (D^\lambda\phi)^\dagger\tau_a\phi\right],$$

$$(2.10b) \quad j_{(1)}^{\ \lambda} = g_1\left[Y\overline{\psi}_{Lm_i}\gamma^\lambda\psi_L{}^{m_i} + Y\overline{\psi}_{Rm_i}\gamma^\lambda\psi_R{}^{m_i}\right] + i\frac{g_1}{4\hbar}\left[\phi^\dagger D^\lambda\phi - (D^\lambda\phi)^\dagger\phi\right]$$

and in the Higgs-field equation

$$(2.11) \qquad D_\mu D^\mu\phi + \mu^2\phi + \frac{\lambda}{6}(\phi^\dagger\phi)\phi = -2k\overline{\psi}_{Rm_i}\hat{x}^{m_i}{}_{n_j}\psi_L{}^{n_j}.$$

Obviously the current densities separate into two gauge-covariant parts $j^{\lambda}_{(2)a}(\psi)$ and $j^{\lambda}_{(2)a}(\phi)$ as well as $j^{\lambda}_{(1)}(\psi)$ and $j^{\lambda}_{(1)}(\phi)$. In a similar way the gauge-invariant canonical energy-momentum tensor consists of three gauge-invariant parts:

$$T_{\lambda}{}^{\mu} = T_{\lambda}{}^{\mu}(\psi) + T_{\lambda}{}^{\mu}(F) + T_{\lambda}{}^{\mu}(\phi) \tag{2.12}$$

with

$$T_{\lambda}{}^{\mu}(\psi) = i\frac{\hbar}{2}\left[\overline{\psi}_{Lm_i}\gamma^{\mu}D_{\lambda}\psi_L{}^{m_i} + \overline{\psi}_{Rm_i}\gamma^{\mu}D_{\lambda}\psi_R{}^{m_i}\right] + h.c., \tag{2.12a}$$

$$\begin{aligned} T_{\lambda}{}^{\mu}(F) = -\frac{\hbar}{4\pi}\Big[&(F^a_{(2)\lambda\nu}F^{\mu\nu}_{(2)a} - \frac{1}{4}\delta_{\lambda}{}^{\mu}F^a_{(2)\alpha\beta}F^{\alpha\beta}_{(2)a}) + \\ &+(F_{(1)\lambda\nu}F^{\mu\nu}_{(1)} - \frac{1}{4}\delta_{\lambda}{}^{\mu}F_{(1)\alpha\beta}F^{\alpha\beta}_{(1)})\Big] \end{aligned} \tag{2.12b}$$

and

$$\begin{aligned} T_{\lambda}{}^{\mu}(\phi) = \frac{1}{2}\Big[&(D_{\lambda}\phi)^{\dagger}D^{\mu}\phi + (D^{\mu}\phi)^{\dagger}D_{\lambda}\phi - \\ &-\delta_{\lambda}{}^{\mu}\left\{(D_{\alpha}\phi)^{\dagger}D^{\alpha}\phi - \mu^2\phi^{\dagger}\phi - \frac{\lambda}{12}(\phi^{\dagger}\phi)^2\right\}\Big]. \end{aligned} \tag{2.12c}$$

With respect to the field-equations the conservation laws for energy and momentum of the whole system of fields are valid:

$$\partial_{\mu}T_{\lambda}{}^{\mu} = 0. \tag{2.13}$$

In view of analyzing the interaction caused by the Higgs-field we investigate at first the equation of motion for the expectation value of the 4-momentum of the fermionic matter fields (ψ-fields) and the gauge-fields (F-fields). From (2.12) and (2.13) one finds immediately under neglection of surface-integrals in the space-like infinity:

$$\partial_0\int\left[T_{\lambda}{}^{0}(\psi) + T_{\lambda}{}^{0}(F)\right]d^3x = -\int\partial_{\mu}T_{\lambda}{}^{\mu}(\phi)d^3x. \tag{2.14}$$

Insertion of ${T_\lambda}^\mu(\phi)$ according to (2.12c) and elimination of the second derivatives of the Higgs-field by the field-equation (2.11) results with the use of the definitions of the field-strengths $F_{(1)\mu\nu}$ and $F^a_{(2)\mu\nu}$ in:

$$\frac{\partial}{\partial t}\int\Big[{T_\lambda}^0(\psi)+{T_\lambda}^0(F)\Big]\,d^3x=$$

$$=k\int\Big[(D_\lambda\phi)^\dagger\overline{\psi}_{Rm_i}\hat{x}^{m_i}_{\ \ n_j}{\psi_L}^{n_j}+\overline{\psi}_{Lm_i}\hat{x}^{\dagger m_i}_{\ \ \ n_j}{\psi_R}^{n_j}D_\lambda\phi\Big]\,d^3x+$$

$$+\frac{i}{2}\int\Big[g_2F^a_{(2)\mu\lambda}\Big\{\phi^\dagger\tau_aD^\mu\phi-(D^\mu\phi)^\dagger\tau_a\phi\Big\}+$$

$$+\frac{1}{2}g_1F_{(1)\mu\lambda}\Big\{\phi^\dagger D^\mu\phi-(D^\mu\phi)^\dagger\phi\Big\}\Big]\,d^3x. \tag{2.15}$$

The right hand side represents the expectation value of the 4-force, which changes the 4-momentum of the ψ-fields and of the F-fields with time. However, the latter expression can be rewritten with the use of the field-equations (2.9a) and (2.9b) as follows:

$$\partial_\mu{T_\lambda}^\mu(F)=\hbar\Big[F^a_{(2)\mu\lambda}\Big\{j_{(2)a}^{\ \mu}(\psi)+j_{(2)a}^{\ \mu}(\phi)\Big\}+$$

$$+F_{(1)\mu\lambda}\Big\{j_{(1)}^{\ \mu}(\psi)+j_{(1)}^{\ \mu}(\phi)\Big\}\ \Big]. \tag{2.16}$$

Herewith one obtains instead of (2.15):

$$\frac{\partial}{\partial t}\int{T_\lambda}^0(\psi)d^3x=\int\hbar\Big[F^a_{(2)\mu\lambda}j_{(2)a}^{\ \mu}(\psi)+F_{(1)\mu\lambda}j^\mu_{(1)}(\psi)\Big]\,d^3x+$$

$$+k\int\Big[(D_\lambda\phi)^\dagger\overline{\psi}_{Rm_i}\hat{x}^{m_i}_{\ \ n_j}\psi_L^{n_j}+\overline{\psi}_{Lm_i}\hat{x}^{\dagger m_i}_{\ \ \ n_j}\psi_R^{n_j}D_\lambda\phi\Big]\,d^3x, \tag{2.17}$$

where on the right hand side we have the 4-force of the gauge-fields and the Higgs-field, both acting on the matter field and changing its 4-momentum. Evidently, the gauge-field strengths couple to the gauge-currents $j_{(2)a}^{\ \mu}(\psi)$ and $j_{(1)}^{\ \mu}(\psi)$, i.e. to the gauge-coupling constants g_1 and g_2 according to (2.10a) and (2.10b), whereas the Higgs-field strength (gradient of the Higgs-field) couples to the fermionic mass-parameter k only (c.f. [4]). This fact points to a gravitational action of the scalar Higgs-field.

3. Field-Equations of Higgs-Gravity.

For demonstrating the gravitational interaction explicitly we perform at first the spontaneous symmetry breaking, because in the case of a scalar gravity only massive particles should interact. [2] For this $\mu^2 < 0$ must be valid, and according to (2.11) and (2.12c) the ground-state ϕ_0 of the Higgs-field is defined by

$$\phi_0{}^\dagger \phi_0 = v^2 = \frac{-6\mu^2}{\lambda}, \tag{3.1}$$

which we resolve as

$$\phi_0 = vN \tag{3.2}$$

with

$$N^\dagger N = 1, \quad \partial_\lambda N = 0. \tag{3.2a}$$

The general Higgs-field ϕ is different from (3.2) by a local unitary transformation:

$$\phi = \rho U N, \quad U^\dagger U = 1 \tag{3.3}$$

with

$$\phi^\dagger \phi = \rho^2, \quad \rho = v(1 + \varphi), \tag{3.3a}$$

where φ represents the real valued excited Higgs-field. Now we use the possibility of a unitary gauge transformation which is inverse to (3.3):

$$\phi' = U^{-1}\phi, \quad \psi' = U^{-1}\psi, \quad \mathcal{F}'_{\mu\nu} = U^{-1}\mathcal{F}_{\mu\nu}U, \tag{3.4}$$

so that

$$\phi' = \rho N \tag{3.4a}$$

is valid, and perform in the following all calculations in the gauge (3.4) (unitary gauge).

[2] The only possible source of a classical scalar gravity is the trace of the energy-momentum tensor.

Using (3.2) up to (3.4a) the field equations (2.8a) through (2.11) take the form, avoiding the strokes introduced in (3.4):

$$i\gamma^\mu D_\mu \psi_L^{m_i} - \frac{1}{\hbar}(1+\varphi)\hat{m}^{m_i}_{\ n_j}\psi_R^{\ n_j} = 0, \tag{3.5a}$$

$$i\gamma^\mu D_\mu \psi_R^{m_i} - \frac{1}{\hbar}(1+\varphi)\hat{m}^{m_i}_{\ n_j}\psi_L^{\ n_j} = 0, \tag{3.5b}$$

$$D_\mu F^{\mu\lambda}_{(2)a} + \frac{1}{\hbar^2}(1+\varphi)^2\left[M^2_{(2)ab}W^{b\lambda} + M^2_{(1,2)a}B^\lambda\right] = 4\pi j^{\ \lambda}_{(2)a}(\psi), \tag{3.6a}$$

$$\partial_\mu F^{\mu\lambda}_{(1)} + \frac{1}{\hbar^2}(1+\varphi)^2\left[M^2_{(1,2)a}W^{a\lambda} + M^2_{(1)}B^\lambda\right] = 4\pi j^{\ \lambda}_{(1)}(\psi), \tag{3.6b}$$

$$\partial_\mu\partial^\mu\varphi + \frac{M^2}{\hbar^2}\varphi + \frac{1}{2}\frac{M^2}{\hbar^2}(3\varphi^2+\varphi^3) =$$

$$= -\frac{1}{v^2}\left[\overline{\psi}_{Lm_i}\hat{m}^{m_i}_{\ n_j}\psi_R^{n_j} + \overline{\psi}_{Rm_i}\hat{m}^{m_i}_{\ n_j}\psi_L^{n_j} - \right.$$

$$\left. - \frac{1}{4\pi\hbar}\left\{M^2_{(2)ab}W^a_{\ \lambda}W^{b\lambda} + 2M^2_{(1,2)a}W^a_{\ \lambda}B^\lambda + M^2_{(1)}B_\lambda B^\lambda\right\}(1+\varphi)\right], \tag{3.7}$$

wherein

$$M^2 = -2\mu^2\hbar^2, \quad (\mu^2 < 0) \tag{3.7a}$$

is the square of the mass of the Higgs-field (φ-field) and

$$\hat{m}^{m_i}_{\ n_j} = kv(N^\dagger\hat{x}^{m_i}_{\ n_j} + \hat{x}^{\dagger m_i}_{\ \ n_j}N) \tag{3.8}$$

is the mass-matrix of the fermionic ψ-fields, which must be adjusted to the observed mass-values of the fermions. The matrices of the mass-squares of the gauge fields are defined by

$$M^2_{(2)ab} = 4\pi\hbar v^2 g_2^2 N^\dagger\tau_{(a}\tau_{b)}N = M^2_W\delta_{ab}, \tag{3.9a}$$

$$(3.9b) \qquad M^2_{(1,2)a} = 4\pi\hbar v^2 g_1 g_2 \frac{1}{2} N^\dagger \tau_a N = -M_W^2 \frac{g_1}{g_2} \delta_a{}^3,$$

$$(3.9c) \qquad M^2_{(1)} = \pi\hbar v^2 g_1^2 = M_W^2 (\frac{g_1}{g_2})^2,$$

where $N = \binom{0}{1}$ is chosen and

$$(3.10) \qquad M_W = \sqrt{\pi\hbar} v g_2.$$

Diagonalization of (3.9a) up to (3.9c) yields the four eigenvalues:

$$(3.11) \qquad M_W^2; \quad M_W^2; \quad M_Z^2 = \pi\hbar v^2 (g_1^2 + g_2^2); \quad 0$$

with the corresponding eigenvectors:

$$(3.11a) \qquad W^1{}_\lambda; \quad W^2{}_\lambda; \quad Z_\lambda = c_W W^3{}_\lambda - s_W B_\lambda; \quad A_\lambda = s_W W^3{}_\lambda + c_W B_\lambda,$$

wherein $c_W = \cos\theta_W$ and $s_W = \sin\theta_W$ (θ_W Weinberg-angle). The field-strengths belonging to (3.11a) are given by:

$$F_{(W^1)}^{\mu\lambda} = F_{(2)}^{1\mu\lambda}; \quad F_{(W^2)}^{\mu\lambda} = F_{(2)}^{2\mu\lambda};$$

$$F_{(Z)}^{\mu\lambda} = c_W F_{(2)}^{3\mu\lambda} - s_W F_{(1)}^{\mu\lambda};$$

$$(3.12) \qquad F_{(A)}^{\mu\lambda} = s_W F_{(2)}^{3\mu\lambda} + c_W F_{(1)}^{\mu\lambda}.$$

Herewith we obtain from (3.6a) and (3.6b) in view of (3.9a) through (3.11) the gauge-field equations: [3]

$$(3.13a) \qquad D_\mu F_{(W^{1,2})}^{\mu\lambda} + (1+\varphi)^2 \left(\frac{M_W}{\hbar}\right)^2 W^{1,2\lambda} = 4\pi j_{(2)}^{1,2\lambda}(\psi),$$

$$(3.13b) \qquad D_\mu F_{(Z)}^{\mu\lambda} + (1+\varphi)^2 \left(\frac{M_Z}{\hbar}\right)^2 Z^\lambda = 4\pi j_{(Z)}^{\lambda}(\psi),$$

[3]The covariant derivative in (3.13a, b, c) is defined by the covariant derivative of the right hand side of (3.12) according to (2.9a).

(3.13c) $$D_\mu F^{\mu\lambda}_{(A)} = 4\pi j^{\lambda}_{(A)}(\psi)$$

with the matter current densities corresponding to (3.12):

(3.14a) $$j^{\lambda}_{(Z)}(\psi) = c_W j^{3\lambda}_{(2)}(\psi) - s_W j^{\lambda}_{(1)}(\psi),$$

(3.14b) $$j^{\lambda}_{(A)}(\psi) = s_W j^{3\lambda}_{(2)}(\psi) + c_W j^{\lambda}_{(1)}(\psi).$$

In the same way we find from (3.7) for the Higgs-field φ:

$$\partial_\mu \partial^\mu \varphi + \frac{M^2}{\hbar^2}\varphi + \frac{1}{2}\frac{M^2}{\hbar^2}(3\varphi^2 + \varphi^3) =$$

$$= -\frac{1}{v^2}\left[\overline{\psi}_{Lm_i}\hat{m}^{m_i}{}_{n_j}\psi_R^{n_j} + \overline{\psi}_{Rm_i}\hat{m}^{m_i}{}_{n_j}\psi_L^{n_j} - \right.$$

(3.15) $$\left. - \frac{1}{4\pi\hbar}\left\{M_W^2(W^1_\lambda W^{1\lambda} + W^2_\lambda W^{2\lambda}) + M_Z^2 Z_\lambda Z^\lambda\right\}(1+\varphi)\right].$$

Obviously, in the field-equations (3.5a), (3.5b), (3.13a) through (3.13c) and (3.15) the Higgs-field φ plays the role of an (attractive) scalar gravitational potential between the massive particles: According to equ. (3.15) the source of φ is the mass of the fermions and of the gauge bosons $W^{1,2}$ and Z, [4] whereby this equation linearized with respect to φ is a potential equation of Yukawa-type. Accordingly the potential φ has a finite range

(3.16) $$l = \hbar/M$$

given by the mass of the Higgs-particle, and v^{-2} has the meaning of the gravitational constant, so that

(3.17) $$v^{-2} = 4\pi G\gamma$$

is valid, where G is the Newtonian gravitational constant and γ a dimensionless factor, which compares the strength of the Newtonian gravity with that of the Higgs-field and which can be determined only experimentally, see sect.

[4] The second term on the right hand side of equ. (3.15) is positive with respect to the signature of the metric.

5. On the other hand, the gravitational potential φ acts back on the mass of the fermions and of the gauge-bosons according to the field equations (3.5a), (3.5b) and (3.13) through (3.13c). Simultaneously the equivalence between inertial and passive as well as active gravitational mass is guaranteed. This feature results from the fact that by the symmetry breaking only one type of mass is introduced. Evidently, the neutrinos ν_L^i and the photon A do not participate in this gravitational interaction.

4. Gravitational Force and Potential Equation.

At first we consider the potential equation from a more classical standpoint. With respect to the fact of a scalar gravitational interaction we rewrite equation (3.15) with the help of the trace of the energy-momentum tensor, because this should be the only source of a scalar gravitational potential within a Lorentz-covariant theory. From (2.12) and (2.12a) through (2.12c) one finds after symmetry breaking:

$$T_\lambda{}^\mu = T_\lambda{}^\mu(\psi) + T_\lambda{}^\mu(W,Z,A) + T_\lambda{}^\mu(\varphi) \tag{4.1}$$

with $T_\lambda{}^\mu(\psi)$ given by (2.12a) and

$$T_\lambda{}^\mu(W,Z,A) = T_\lambda{}^\mu(F) + \frac{1}{4\pi\hbar}\Big[M_W^2\Big\{(W_\lambda^1 W^{1\mu} + W_\lambda^2 W^{2\mu}) -$$

$$-\frac{1}{2}\delta_\lambda{}^\mu(W_\alpha^1 W^{1\alpha} + W_\alpha^2 W^{2\alpha})\Big\} + M_Z^2\Big\{Z_\lambda Z^\mu - \frac{1}{2}\delta_\lambda{}^\mu Z_\alpha Z^\alpha\Big\}\Big] \tag{4.1a}$$

($T_\lambda{}^\mu(F)$ according to (2.12b)) as well as

$$T_\lambda{}^\mu(\varphi) = v^2\left[\partial_\lambda\varphi\partial^\mu\varphi - \frac{1}{2}\delta_\lambda{}^\mu\left\{\partial_\alpha\varphi\partial^\alpha\varphi + \frac{M^2}{4\hbar^2}(1+\varphi)^2(1-2\varphi-\varphi^2)\right\}\right]. \tag{4.1b}$$

From this it follows immediately using the field equations (3.5a) and (3.5b):

$$T = T_\lambda{}^\lambda = T(\psi) + T(W,Z,A) + T(\varphi) \tag{4.2}$$

with

$$T(\psi) = \left[\overline{\psi}_{Lm_i}\hat{m}^{m_i}{}_{n_j}\psi_R{}^{n_j} + \overline{\psi}_{Rm_i}\hat{m}^{m_i}{}_{n_j}\psi_L{}^{n_j}\right](1+\varphi), \tag{4.2a}$$

$$T(W,Z,A) = T(W,Z) = -\frac{1}{4\pi\hbar}\Big[M_W^2(W_\lambda^1 W^{1\lambda}+$$

$$+W_\lambda^2 W^{2\lambda}) + M_Z^2 Z_\lambda Z^\lambda\Big](1+\varphi)^2 \tag{4.2b}$$

and

$$T(\varphi) = v^2\left[\frac{M^2}{2\hbar^2}(\varphi^4+4\varphi^3+4\varphi^2-1) - \partial_\lambda\varphi\partial^\lambda\varphi\right]. \tag{4.2c}$$

In the appendix it is shown that $T(\psi)$ separates in total analogy to $T(W,Z)$ into the masses of the single fermions:

$$T(\psi) = \sum_i (m_{e^i}\overline{e}_i e^i + m_{u^i}\overline{u}_i u^i + m_{d^i}\overline{d}_i d^i)(1+\varphi). \tag{4.2a'}$$

Comparing (4.2a) and (4.2b) with the right hand side of the Higgs-field equation (3.15) one finds that the source of the potential φ is given by the first two terms of the trace (4.2). In this way we find using (3.17):

$$\partial_\mu\partial^\mu\varphi + \frac{M^2}{\hbar^2}\varphi + \frac{1}{2}\frac{M^2}{\hbar^2}(3\varphi^2+\varphi^3) =$$

$$= -4\pi G\gamma(1+\varphi)^{-1}(T(\psi)+T(W,Z)). \tag{4.3}$$

In the linearized version (with respect to φ) equ. (4.3) represents a potential equation for φ of Yukawa-type with the trace of the energy-momentum tensor of the massive fermions and the massive gauge-bosons $W^{1,2}$ and Z as source.

Finally we investigate the gravitational force caused by the Higgs-field more in detail. Insertion of the symmetry breaking according to (3.1) up to (3.4a) into the first integral of the right hand side of (2.15) yields:

$$K_\lambda = k\Big[(D_\lambda\phi)^\dagger\overline{\psi}_{Rm_i}\hat{x}^{m_i}{}_{n_j}\psi_L{}^{n_j} + \overline{\psi}_{Lm_i}\hat{x}^{\dagger m_i}{}_{n_j}\psi_R{}^{n_j}D_\lambda\phi\Big] =$$

$$= (\overline{\psi}_{Rm_i}\hat{m}^{m_i}{}_{n_j}\psi_L{}^{n_j} + \overline{\psi}_{Lm_i}\hat{m}^{m_i}{}_{n_j}\psi_R{}^{n_j})\partial_\lambda\varphi +$$

$$+v(1+\varphi)\Big[(D_\lambda N)^\dagger k\overline{\psi}_{Rm_i}\hat{x}^{m_i}{}_{n_j}\psi_L{}^{n_j} + k\overline{\psi}_{Lm_i}\hat{x}^{\dagger m_i}{}_{n_j}\psi_R{}^{n_j}D_\lambda N\Big]. \tag{4.4}$$

Substitution of the conglomerate $k\overline{\psi}_{Rm_i}\hat{x}^{m_i}_{\ n_j}\psi_L^{\ n_j}$ by the left hand side of the field equation (2.11) results with the use of (3.3a) and (3.4a) in:

$$K_\lambda = \Big[\overline{\psi}_{Lm_i}\hat{m}^{m_i}_{\ n_j}\psi_R^{\ n_j} + \overline{\psi}_{Rm_i}\hat{m}^{m_i}_{\ n_j}\psi_L^{\ n_j} -$$

$$-\frac{1}{4\pi\hbar}\Big\{M_W^2(W^1_{\ \alpha}W^{1\alpha} + W^2_\alpha W^{2\alpha}) + M_Z^2 Z_\alpha Z^\alpha\Big\}(1+\varphi)\Big]\partial_\lambda\varphi -$$

$$-\frac{1}{4\pi\hbar}\partial_\mu\Big[(1+\varphi)^2\Big\{M_W^2(W^1_\lambda W^{1\mu} + W^2_\lambda W^{2\mu} -$$

$$-\frac{1}{2}\delta_\lambda^{\ \mu}\Big[W^1_\alpha W^{1\alpha} + W^2_\alpha W^{2\alpha}\Big]) + M_Z^2(Z_\lambda Z^\mu - \frac{1}{2}\delta_\lambda^{\ \mu}Z_\alpha Z^\alpha)\ \Big\}\Big] +$$

$$+i\frac{v^2}{2}(1+\varphi)^2\Big[g_2 F^a_{(2)\mu\lambda}\Big\{N^\dagger\tau_a D^\mu N - (D^\mu N)^\dagger\tau_a N\Big\} +$$

$$+ g_1 F_{(1)\mu\lambda}\Big\{N^\dagger D^\mu N - (D^\mu N)^\dagger N\Big\}\Big]. \tag{4.5}$$

By insertion of (4.5) into the right hand side of (2.15) the last brackets of (4.5) and (2.15) cancel out, whereas the second bracket of (4.5) can be combined with $\partial_\mu T_\lambda^{\ \mu}(F)$ to $\partial_\mu T_\lambda^{\ \mu}(W,Z,A)$ according to (4.2b). In this way we obtain neglecting surface integrals in the space-like infinity:

$$\frac{\partial}{\partial t}\int\Big[T_\lambda^{\ 0}(\psi) + T_\lambda^{\ 0}(W,Z,A)\Big]d^3x =$$

$$= \int\Big[\overline{\psi}_{Lm_i}\hat{m}^{m_i}_{\ n_j}\psi_R^{\ n_j} + \overline{\psi}_{Rm_i}\hat{m}^{m_i}_{\ n_j}\psi_L^{\ n_j} -$$

$$-\frac{1}{4\pi\hbar}\Big\{M_W^2(W^1_\alpha W^{1\alpha} + W^2_\alpha W^{2\alpha}) + M_Z^2 Z_\alpha Z^\alpha\Big\}(1+\varphi)\Big]\partial_\lambda\varphi d^3x. \tag{4.6}$$

In total analogy to the procedure yielding the potential equation (4.3) we substitute the bracket of the 4-force in (4.6) by the traces $T(\psi)$ and $T(W,Z)$ given by (4.2a) and (4.2b) respectively; so we find:

$$\frac{\partial}{\partial t}\int\Big[T_\lambda^{\ 0}(\psi) + T_\lambda^{\ 0}(W,Z,A)\Big]d^3x =$$

$$= \int (1+\varphi)^{-1} \Big[T(\psi) + T(W,Z) \Big] \partial_\lambda \varphi d^3x. \tag{4.7}$$

Considering the transition from equ. (2.15) to (2.17) we can express the time derivative of the 4-momentum of the gauge-fields by a 4-force acting on the fermionic matter currents. Restricting this procedure to the massless gauge-field A^λ (photon) we get from (4.7):

$$\frac{\partial}{\partial t} \int \Big[T_\lambda{}^0(\psi) + T_\lambda{}^0(W,Z) \Big] d^3x = \int \hbar F_{(A)\lambda\mu} j^\mu_{(A)}(\psi) d^3x +$$

$$+ \int (1+\varphi)^{-1} \left[T(\psi) + T(W,Z) \right] \partial_\lambda \varphi d^3x. \tag{4.8}$$

Herein the first term of the right hand side describes the 4-force of the massless gauge-boson acting on the matter fields, i.e. the electromagnetic Lorentz-force coupled by the electric charge, see (3.14b):

$$e = s_W g_2 = c_W g_1. \tag{4.8a}$$

The second term (identical with the right hand side of (4.7)) is the attractive gravitational force on the masses of the fermions and of the gauge-bosons $W^{1,2}$ and Z, which are simultaneously the source of the Higgs-potential φ according to (4.3). This behaviour is exactly that of classical gravity, coupling to the mass ($\equiv$ energy) only and not to any charge. However, the qualitative difference with respect to the Newtonian gravity consists besides the non-linear terms in (4.3) in the finite range of φ caused by the Yukawa-term.

5. Final Remarks.

In the end we point to some interesting features of our result. First of all we note, that in view of the right hand side of (4.7) it is appropriate to define

$$\ln(1+\varphi) = \chi \tag{5.1}$$

as new gravitational potential, so that the momentum law reads:

$$\frac{\partial}{\partial t} \int \Big[T_\lambda{}^0(\psi) + T_\lambda{}^0(W,Z,A) \Big] d^3x =$$

$$(5.2) \qquad = \int \left[T(\psi) + T(W,Z)\right] \partial_\lambda \chi d^3x.$$

Then the non-linear terms concerning φ in (4.3) can be expressed by $T(\varphi) \equiv T(\chi)$ according to (4.2c). In this way the field equation for the potential χ (excited Higgs-field) takes the very impressive form:

$$(5.3) \qquad \partial_\mu \partial^\mu e^{2\chi} + \frac{M^2}{\hbar^2} e^{2\chi} = -8\pi G\gamma \left[T(\psi) + T(W,Z) + T(\chi)\right].$$

Equations (5.2) and (5.3) are indeed those of scalar gravity with self-interaction in a natural manner. For the understanding of the Higgs-field it may be of interest, that the structure of equation (5.3) exists already before the symmetry breaking. Considering the trace T of the energy-momentum tensor (2.12) one finds with the use of the field-equations (2.8a), (2.8b) and (2.11):

$$(5.4) \qquad \partial_\mu \partial^\mu (\phi^\dagger \phi) + (\frac{M}{\hbar})^2 (\phi^\dagger \phi) = -2T$$

with $M^2 = -2\mu^2\hbar^2$. Accordingly, the Yukawa-like self-interacting scalar gravity of the Higgs-field is present within the theory from the very beginning. Equation (5.4) possesses an interesting behaviour with respect to the symmetry breaking. From the second term on the left hand side there results in view of (3.1) in the first step a cosmological constant $M^2v^2/\hbar^2$; but this is compensated exactly by the trace of the energy-momentum tensor of the ground-state. In our opinion this is the property of the cosmological constant at all, also in general relativity.

Furthermore we emphasize that the gravitational action of the Higgs-field is not restricted to the Glashow-Salam-Weinberg model, but it is valid in all cases of mass producing by symmetry breaking via the Higgs-mechanism [6], e.g. also in the GUT-model. However, because in (3.16) the mass M is that of the Higgs-particle, the range l of the potential φ should be very short, so that until now no experimental evidence for the Higgs-gravity may exist, at least in the macroscopic limit. For this reason it also appears unprobable, that it has to do something with the non-Newtonian gravity currently discussed as so called fifth force [7].

Finally, the factor γ in (3.17) can be calculated from (3.10) by the use of the mass of the W-bosons and the value of the gauge-coupling constant g_2;

one finds:

$$\gamma = \hbar g_2^2/4GM_W^2 = \frac{1}{2}g_2^2(\frac{M_P}{M_W})^2 = 2\times 10^{32} \tag{5.5}$$

(M_P Planck mass). Consequently, the Higgs-gravity represents a relatively strong scalar gravitational interaction between the massive elementary particles, however with extremely short range and with the essential property of quantizability. If any Higgs-field exists in nature, this type of gravity is present.

The expression (5.5) shows, that in the case of a symmetry breaking where the bosonic mass is of the order of the Planck mass, the Higgs-gravity approaches the Newtonian gravity, if the mass of the Higgs-particle is sufficiently small. In this connection the question arises, following Einstein's idea of relativity of inertia, if it is possible to construct a tensorial quantum theory of gravity with the use of the Higgs-mechanism, leading at last to Einstein's gravitational theory in the classical macroscopic limit.

Appendix: In order to show the separation of $T(\psi)$ into the single fermionic masses it is necessary to specify the fermionic mass-matrix as follows (without suppression of the $SU(2)$ indices $I, J, ...$): [5]

$$(A1) \qquad \hat{m}^{m_i}_{\ n_j} = \hat{m}^{Im_i}_{\ Jn_j} = \sum_k m_{Im_k} \delta^I_{\ J} \delta^m_{\ n} (U_{(c)}^{\ I_m})^i_k (U_{(c)}^{\ I_m})^{-1k}_{\ \ j}$$

where

$$(A2) \qquad (U^{I_m}_{(c)})^i_k = \begin{cases} U_{(c)k}^{\ i} & \text{if} \quad (I,m) = (d', q), \\ \delta^i_{\ k} & \text{otherwise} \end{cases}$$

with the Cabibbo-matrix $U_{(c)k}^{\ i}$ according to (2.3b). Insertion of (A1) into the right hand side of (4.2a) yields:

$$\overline{\psi}_{Lm_i} \hat{m}^{m_i}_{\ n_j} \psi_R^{\ n_j} + \overline{\psi}_{Rm_i} \hat{m}^{m_i}_{\ n_j} \psi_L^{\ n_j} =$$

$$(A3)$$
$$= \sum_i \left[m_{e^i} (\overline{e}_{R_i} e_L^{\ i} + \overline{e}_{L_i} e_R^{\ i}) + m_{u^i} (\overline{u}_{R_i} u_L^{\ i} + \overline{u}_{L_i} u_R^{\ i}) + m_{d^i} (\overline{d}_{R_i} d_L^{\ i} + \overline{d}_{L_i} d_R^{\ i}) \right],$$

which immediately goes over into the expression (4.2a').

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