

Higgs-Field Gravity.

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Summary

It is shown that any excited Higgs-field mediates an attractive scalar gravitational interaction of Yukawa-type between the elementary particles, which become massive by the ground-state of the Higgs-field.

1. Introduction.

Until now the origin of the mass of the elementary particles is unclear. Usually mass is introduced by the interaction with the Higgs-field; however in this way the mass is not explained, but only reduced to the parameters of the Higgs-potential, whereby the physical meaning of the Higgs-field and its potential remains non-understood. We want to give here a contribution for its interpretation.

There exists an old idea of Einstein, the so called "principle of relativity of inertia" according to which mass should be produced by the interaction with the gravitational field [1]. Einstein argued that the inertial mass is only a measure for the resistance of a particle against the relative acceleration with respect to other particles; therefore within a consequent theory of relativity the mass of a particle should be originated by the interaction with all other particles of the Universe (Mach's principle), whereby this interaction should be the gravitational one which couples to all particles, i.e. to their masses or energies. He postulated even that the value of the mass of a particle should go to zero, if one puts the particle in an infinite distance of all other ones.

This fascinating idea was not very successful in Einstein's theory of gravity, i.e. general relativity, although it has caused, that Einstein introduced the cosmological constant in order to construct a cosmological model with finite space, and that Brans and Dicke developed their scalar-tensor-theory [2]. But an explanation of the mass does not follow from it until now.

In this paper we will show, that the successful Higgs-field mechanism lies in the direction of Einstein's idea of producing mass by gravitational interaction; we find, that the Higgs-field as source of the inertial mass of the elementary particles has to do something with gravity [3], i.e. it mediates a scalar gravitational interaction between massive particles, however of Yukawa type. This results from the fact, that the Higgs-field itself becomes massive after symmetry breaking. On the other hand an estimation of the coupling

constants shows that it may be unprobable that this Higgs-field gravity can be identified with any experimental evidence. Perhaps its applicability lies beyond the scope of the present experimental experiences.

2. Gravitational force and potential equation.

We perform our calculations in full generality with the use of an U(N) model and start from the Lagrange density of fermionic fields coupled with the Higgs-field both belonging to the localized group U(N) ($c = 1, \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$):

$$(1) \quad L = \frac{\hbar}{2} i \bar{\psi} \gamma^\mu D_\mu \psi + h.c. - \frac{\hbar}{16\pi} F^a{}_{\lambda\mu} F_a{}^{\lambda\mu} + \\ + \frac{1}{2} (D_\mu \phi)^\dagger D^\mu \phi - \frac{\mu^2}{2} \phi^\dagger \phi - \frac{\lambda}{4!} (\phi^\dagger \phi)^2 - k \bar{\psi} \phi^\dagger \hat{x} \psi + h.c.$$

(μ^2, λ, k are real parameters of the Higgs-potential). Herein D_μ represents the covariant derivative with respect to the localized group U(N)

$$(1a) \quad D_\mu = \partial_\mu + ig A_\mu$$

(g gauge coupling constant, $A_\mu = A_\mu^a \tau_a$ gauge potentials, τ_a generators of the group U(N)) and the gauge field strength $F_{\mu\nu}$ is determined by its commutator ($F_{\mu\nu} = (1/ig) [D_\mu, D_\nu] = F_{\mu\nu}^a \tau_a$); furthermore \hat{x} is the Yukawa coupling-matrix. For the case of applying the Lagrange density (1) to a special model, as e.g. the Glashow-Salam-Weinberg model or even the GUT-model, the wave function ψ , the generators τ_a , the Higgs-field ϕ and the coupling matrix \hat{x} must be specified explicitly [4].

From (1) we get immediately the field equations for the spinorial matter fields (ψ -fields):

$$(2) \quad i \gamma^\mu D_\mu \psi - \frac{k}{\hbar} (\phi^\dagger \hat{x} + \hat{x}^\dagger \phi) \psi = 0,$$

the Higgs-field ϕ

$$(3) \quad D^\mu D_\mu \phi + \mu^2 \phi + \frac{\lambda}{3!} (\phi^\dagger \phi) \phi = -2k \bar{\psi} \hat{x} \psi$$

and the gauge-fields $F^{a\mu\lambda}$

$$(4) \quad \partial_\mu F^{a\mu\lambda} + ig f_{bc}^a A^{b\mu} F_\mu^{c\lambda} = 4\pi j^{a\lambda}$$

with the gauge-current density

$$(4a) \quad j^{a\lambda} = g(\bar{\psi}\gamma^\lambda\tau^a\psi + \frac{i}{2\hbar} [\phi^\dagger\tau^a D^\lambda\phi - (D^\lambda\phi)^\dagger\tau^a\phi]).$$

Herein f_{bc}^a are the totally skew symmetric structure constants of the group U(N). The gauge invariant canonical energy momentum tensor reads with the use of (2)

$$(5) \quad T_\lambda^\mu = \frac{i\hbar}{2} [\bar{\psi}\gamma^\mu D_\lambda\psi - (\overline{D_\lambda\psi})\gamma^\mu\psi] - \\ - \frac{\hbar}{4\pi} \left[F_{\lambda\nu}^\alpha F_a^{\mu\nu} - \frac{1}{4}\delta_\lambda^\mu F_{\alpha\beta}^a F_a^{\alpha\beta} \right] + \\ + \frac{1}{2} \left[(D_\lambda\phi)^\dagger D^\mu\phi + (D^\mu\phi)^\dagger D_\lambda\phi - \right. \\ \left. - \delta_\lambda^\mu \{ (D_\alpha\phi)^\dagger D^\alpha\phi - \mu^2\phi^\dagger\phi - \frac{2\lambda}{4!}(\phi^\dagger\phi)^2 \} \right]$$

and fulfils the conservation law

$$(6) \quad \partial_\mu T_\lambda^\mu = 0.$$

Obviously, the current-density (4a) has a gauge-covariant matter-field and Higgs-field part, i.e. $j^{a\lambda}(\psi)$ and $j^{a\lambda}(\phi)$ respectively, whereas the energy-momentum tensor (5) consists of a sum of three gauge-invariant parts:

$$(7) \quad T_\lambda^\mu = T_\lambda^\mu(\psi) + T_\lambda^\mu(F) + T_\lambda^\mu(\phi),$$

represented by the brackets on the right hand side of equ. (5).

In view of analyzing the interaction caused by the Higgs-field we investigate at first the equation of motion for the expectation value of the 4-momentum of the matter fields and the gauge-fields. From (6) and (7) one finds under neglection of surface-integrals in the space-like infinity:

$$(8) \quad \partial_0 \int [T_\lambda^0(\psi) + T_\lambda^0(F)] d^3x = - \int \partial_\mu T_\lambda^\mu(\phi) d^3x.$$

Insertion of $T_\lambda^\mu(\phi)$ according to (5) and elimination of the second derivatives of the Higgs-field by the field-equations (3) results in:

$$\begin{aligned}
& \frac{\partial}{\partial t} \int [T_\lambda^0(\psi) + T_\lambda^0(F)] d^3x = \\
(9) \quad & = k \int \bar{\psi} [(D_\lambda \phi)^\dagger \hat{x} + \hat{x}^\dagger (D_\lambda \phi)] \psi d^3x + \\
& + \frac{ig}{2} \int F_{\mu\lambda}^a [\phi^\dagger \tau_a D^\mu \phi - (D^\mu \phi)^\dagger \tau_a \phi] d^3x.
\end{aligned}$$

The right hand side represents the expectation value of the 4-force, which causes the change of the 4-momentum of the ψ -fields and the F-fields with time. However, the last expression can be rewritten with the use of the field-equations (4) as follows:

$$(9a) \quad \partial_\mu T_\lambda^\mu(F) = \hbar F_{\mu\lambda}^a (j_a^\mu(\psi) + j_a^\mu(\phi)).$$

Herewith one obtains instead of (9):

$$\begin{aligned}
(10) \quad & \frac{\partial}{\partial t} \int T_\lambda^0(\psi) d^3x = \int \hbar F_{\lambda\mu}^a j_a^\mu(\psi) d^3x + \\
& + k \int \bar{\psi} [(D_\lambda \phi)^\dagger \hat{x} + \hat{x}^\dagger (D_\lambda \phi)] \psi d^3x,
\end{aligned}$$

where on the right hand side we have the 4-force of the gauge-field and the Higgs-field, both acting on the matter-field. Evidently, the gauge-field strength couples to the gauge-currents, i.e. to the gauge-coupling constant g according to (4a), whereas the Higgs-field strength (gradient of the Higgs-field) couples to the fermionic mass-parameter k (c.f. [5]). This fact points to a gravitational action of the scalar Higgs-field.

a) Gravitational interaction on the level of the field-equations.

For demonstrating the gravitational interaction explicitly we perform at first the spontaneous symmetry breaking, because in the case of a scalar gravity

only massive particles should interact.¹ For this $\mu^2 < 0$ must be valid, and according to (3) and (5) the ground state ϕ_0 of the Higgs-field is defined by

$$(11) \quad \phi_0^\dagger \phi_0 = v^2 = \frac{-6\mu^2}{\lambda},$$

which we resolve as

$$(12) \quad \phi_0 = vN$$

with

$$(12a) \quad N^\dagger N = 1, \quad \partial_\lambda N \equiv 0.$$

The general Higgs-field ϕ is different from (12) by a local unitary transformation:

$$(13) \quad \phi = \rho U N, \quad U^\dagger U = 1$$

with

$$(13a) \quad \phi^\dagger \phi = \rho^2, \quad \rho = v^\dagger \eta,$$

where η represents the real valued excited Higgs-field.

Now we use the possibility of a unitary gauge transformation which is inverse to (13):

$$\phi' = U^{-1} \phi, \quad \psi' = U^{-1} \psi,$$

$$(14) \quad F'_{\mu\nu} = U^{-1} F_{\mu\nu} U,$$

so that

$$(14a) \quad \phi' = \rho N,$$

and perform in the following all calculations in the gauge (14) (unitary gauge). For this we note, that in the case of the symmetry breaking of the group G

$$(15) \quad G \rightarrow \tilde{G},$$

¹The only possible source of a scalar gravity is the trace of the energy momentum tensor.

where \tilde{G} represents the rest-symmetry group, we decompose the unitary transformation:

$$(15a) \quad U = \hat{U} \cdot \tilde{U}, \quad \tilde{U} \in \tilde{G}, \quad \hat{U} \in G/\tilde{G}$$

with the isotropy property ($\tau_{\hat{a}}$ generators of the unbroken symmetry):

$$(16) \quad \tilde{U}N = e^{i\lambda^{\hat{a}}\tau_{\hat{a}}}N = N,$$

so that

$$(17) \quad \tau_{\hat{a}}N = 0$$

is valid. For \hat{U} we write $\hat{U} = e^{i\lambda^{\hat{a}}\tau_{\hat{a}}}$, where $\tau_{\hat{a}}$ are the generators of the broken symmetry.

Using (12) up to (17) the field-equations (2) through (4) take the form, avoiding the strokes introduced in (14):

$$(18) \quad i\gamma^\mu D_\mu \psi - \frac{\hat{m}}{\hbar}(1 + \varphi)\psi = 0,$$

$$(19) \quad \begin{aligned} & \partial_\mu F_a^{\mu\lambda} + igf_{abc}A^{b\mu}F_\mu^{\lambda c} + \\ & + \frac{1}{\hbar^2}M_{ab}^2(1 + \varphi)^2 A^{b\lambda} = 4\pi j_a^\lambda(\psi), \end{aligned}$$

$$\partial^\mu \partial_\mu \varphi + \frac{M^2}{\hbar^2}\varphi + \frac{1}{2}\frac{M^2}{\hbar^2}(3\varphi^2 + \varphi^3) =$$

$$(20) \quad -\frac{1}{v^2} \left[\bar{\psi} \hat{m} \psi - \frac{1}{4\pi\hbar} M_{ab}^2 A_\lambda^a A^{b\lambda} (1 + \varphi) \right],$$

wherein $\varphi = \eta/v$ represents the excited Higgs-field and

$$(18a) \quad \hat{m} = kv(N^\dagger \hat{x} + \hat{x}^\dagger N)$$

is the mass-matrix of the matter-field (ψ -field),

$$(19a) \quad M_{ab}^2 = M_{\hat{a}\hat{b}}^2 = 4\pi\hbar g^2 v^2 N^\dagger \tau_{(\hat{a}} \tau_{\hat{b})} N$$

the symmetric matrix of the mass-square of the gauge-fields ($A_{\mu}^{\hat{a}}$ -fields) and

$$(20a) \quad M^2 = -2\mu^2\hbar^2, \quad (\mu^2 < 0)$$

is the square of the mass of Higgs-field (φ -field). Obviously in the field-equations (18) up to (20) the Higgs-field φ plays the role of an attractive scalar gravitational potential between the massive particles: According to equ. (20) the source of φ is the mass of the fermions and of the gauge-bosons,² whereby this equation linearized with respect to φ is a potential equation of Yukawa-type. Accordingly the potential φ has a finite range

$$(21) \quad l = \hbar/M$$

given by the mass of the Higgs-particle and v^{-2} has the meaning of the gravitational constant, so that

$$(22) \quad v^{-2} = 4\pi G\gamma$$

is valid, where G is the Newtonian gravitational constant and γ a dimensionless factor, which compares the strength of the Newtonian gravity with that of the Higgs-field and which can be determined only experimentally, see sect. 3. On the other hand, the gravitational potential φ acts back on the mass of the fermions and of the gauge-bosons according to the field equations (18) and (19). Simultaneously the equivalence between inertial and passive as well as active gravitational mass is guaranteed. This feature results from the fact that by the symmetry breaking only one type of mass is introduced.

b) Gravitational interaction on the level of the momentum law.

At first we consider the potential equation from a more classical standpoint. With respect to the fact of a scalar gravitational interaction we rewrite equation (20) with the help of the trace of the energy-momentum tensor, because this should be the only source of a scalar gravitational potential within a

²The second term in the bracket on the right hand side of equ. (20) is positive with respect to the signature of the metric.

Lorentz-covariant theory. From (5) one finds after symmetry breaking in analogy to (7):

$$(23a) \quad T_\lambda^\mu(\psi) = \frac{i\hbar}{2} [\bar{\psi}\gamma^\mu D_\lambda\psi - (\overline{D_\lambda\psi})\gamma^\mu\psi],$$

$$(23b) \quad T_\lambda^\mu(A) = -\frac{\hbar}{4\pi}(F_{\lambda\nu}^a F_a^{\mu\nu} - \frac{1}{4}\delta_\lambda^\mu F_{\alpha\beta}^a F_a^{\alpha\beta}) + \\ + \frac{1}{4\pi\hbar}(1+\varphi)^2 M_{ab}^2 (A_\lambda^a A^{b\mu} - \frac{1}{2}\delta_\lambda^\mu A_\nu^a A^{b\nu}),$$

$$(23c) \quad T_\lambda^\mu(\varphi) = v^2 \left[\partial_\lambda\varphi\partial^\mu\varphi - \frac{1}{2}\delta_\lambda^\mu \left\{ \partial_\alpha\varphi\partial^\alpha\varphi + \right. \right. \\ \left. \left. + \frac{M^2}{4\hbar^2}(1+\varphi)^2(1-2\varphi-\varphi^2) \right\} \right].$$

From this it follows immediately using the field equation (18):

$$(23d) \quad T = T_\lambda^\lambda = \bar{\psi}\hat{m}\psi(1+\varphi) - \frac{1}{4\pi\hbar}M_{ab}^2 A_\lambda^a A^{b\lambda}(1+\varphi)^2 + \\ + v^2 \left[\frac{M^2}{2\hbar^2}(\varphi^4 + 4\varphi^3 + 4\varphi^2 - 1) - \partial_\lambda\varphi\partial^\lambda\varphi \right].$$

The comparison with equ. (20) shows, that the source of the potential φ is given by the first two terms of (23d), i.e. by $T(\psi)$ and $T(A)$ as expected. In this way we obtain as potential equation using (22):

$$(24) \quad \partial^\mu\partial_\mu\varphi + \frac{M^2}{\hbar^2}\varphi + \frac{1}{2}\frac{M^2}{\hbar^2}(3\varphi^2 + \varphi^3) = \\ = -4\pi G\gamma(1+\varphi)^{-1}(T(\psi) + T(A)).$$

In the linearized version (with respect to φ) equ. (24) represents a potential equation for φ of Yukawa-type with the trace of the energy-momentum tensor of the massive fermions and the massive gauge-bosons as source.

Finally we investigate the gravitational force caused by the Higgs-field more in detail. Insertion of the symmetry breaking according to (12) up to (17) into the first integral of the right-hand side of (9) yields:

$$\begin{aligned}
K_\lambda &= k\bar{\psi} \left[(D_\lambda\phi)^\dagger \hat{x} + \hat{x}^\dagger (D_\lambda\phi) \right] \psi = \\
(25) \quad &= \bar{\psi} \hat{m} \psi \partial_\lambda \varphi + v(1 + \varphi) \left[(D_\lambda N)^\dagger k\bar{\psi} \hat{x} \psi + k\bar{\psi} \hat{x}^\dagger \psi D_\lambda N \right].
\end{aligned}$$

Substitution of the conglomerate $k\bar{\psi} \hat{x} \psi$ by the left hand side of the field-equation (3) results with the use of (13a) and (14a) in:

$$\begin{aligned}
K_\lambda &= \left[\bar{\psi} \hat{m} \psi - \frac{1}{4\pi\hbar} M_{ab}^2 A_\mu^a A^{b\mu} (1 + \varphi) \right] \partial_\lambda \varphi - \\
&- \frac{1}{4\pi\hbar} \partial_\mu \left[(1 + \varphi)^2 M_{ab}^2 (A_\lambda^a A^{b\mu} - \frac{1}{2} \delta_\lambda^\mu A_\nu^a A^{b\nu}) \right] + \\
(26) \quad &+ \frac{v^2}{2} i g (1 + \varphi)^2 F_{\lambda\mu}^a \left[N^\dagger \tau_a D^\mu N - (D^\mu N)^\dagger \tau_a N \right].
\end{aligned}$$

By insertion of (26) into the right hand side of (9) the last term of (26) drops out against the last term of (9), whereas the second term of (26) can be combined with $\partial_\mu T_\lambda^\mu(F)$ to $\partial_\mu T_\lambda^\mu(A)$ according to (23b). In this way we obtain neglecting surface integrals in the space-like infinity:

$$\begin{aligned}
\frac{\partial}{\partial t} \int [T_\lambda^0(\psi) + T_\lambda^0(A)] d^3x &= \int [\bar{\psi} \hat{m} \psi - \\
(27) \quad &- \frac{1}{4\pi\hbar} M_{ab}^2 A_\mu^a A^{b\mu} (1 + \varphi)] \partial_\lambda \varphi d^3x.
\end{aligned}$$

In total analogy to the procedure yielding the potential equation (24) we substitute the bracket of the 4-force in (27) by the traces $T(\psi)$ and $T(A)$ given by (23d):

$$\frac{\partial}{\partial t} \int [T_\lambda^0(\psi) + T_\lambda^0(A)] d^3x =$$

$$(28) \quad = \int (1 + \varphi)^{-1} [T(\psi) + T(A)] \partial_\lambda \varphi d^3x.$$

Considering the transition from equ. (9) to (10) we can express the time-derivative of the 4-momentum of the gauge-fields by a 4-force acting on the matter currents. Restricting this procedure to the massless gauge-fields we get from (28):

$$(29) \quad \begin{aligned} & \frac{\partial}{\partial t} \int [T_\lambda^0(\psi) + T_\lambda^0(A_\sigma^{\hat{a}})] d^3x = \\ & = \int \hbar F_{\lambda\mu}^{\hat{a}} j_{\hat{a}}^\mu(\psi) d^3x + \\ & + \int (1 + \varphi)^{-1} [T(\psi) + T(A_\sigma^{\hat{a}})] \partial_\lambda \varphi d^3x. \end{aligned}$$

Herein the first term of the right hand side describes the 4-force of the massless gauge-bosons acting on the matter-field coupled by the gauge-coupling constant g , see (4a), whereas the second term (identical with the right hand side of (28)) is the attractive gravitational force of the Higgs-field φ acting on the masses of the fermions and of the gauge-bosons, which are simultaneously the source of the Higgs-potential φ according to (24). This behaviour is exactly that of classical gravity, coupling to the mass (\equiv energy) only and not to any charge. However the qualitative difference with respect to the Newtonian gravity consists besides the non-linear terms in (24) in the finite range of φ caused by the Yukawa term.

3. Final Remarks.

In the end we want to point to some interesting features of our result. First of all we note, that in view of the right hand side of (28) it is appropriate to define

$$(30) \quad \ln(1 + \varphi) = \chi$$

as new gravitational potential, so that the momentum law reads:

$$(31) \quad \frac{\partial}{\partial t} \int [T_\lambda^0(\psi) + T_\lambda^0(A)] d^3x = \int [T(\psi) + T(A)] \partial_\lambda \chi d^3x.$$

Then the non-linear terms concerning φ in (24) can be expressed by $T(\varphi) \equiv T(\chi)$ according to the third term of the right hand side of (23d). In this way the field equation for the potential χ (excited Higgs-field) takes the very impressive form:

$$(32) \quad \partial_\mu \partial^\mu e^{2\chi} + \frac{M^2}{\hbar^2} e^{2\chi} = -8\pi G\gamma [T(\psi) + T(A) + T(\chi)].$$

Equations (31) and (32) are indeed those of scalar gravity with self interaction in a natural manner. For the understanding of the Higgs-field it may be of interest, that the structure of equation (32) exists already before the symmetry breaking. Considering the trace T of the energy momentum tensor (5) one finds with the use of the field-equations (2) and (3):

$$(33) \quad \partial_\mu \partial^\mu (\phi^\dagger \phi) + \frac{M^2}{\hbar^2} (\phi^\dagger \phi) = -2T$$

with $M^2 = -2\mu^2\hbar^2$. Accordingly, the Yukawa-like self-interacting scalar gravity of the Higgs-field is present within the theory from the very beginning. Equation (33) possesses an interesting behaviour with respect to the symmetry breaking. Then from the second term on the left hand side there results in view of (11) in the first step a cosmological constant $M^2 v^2 / \hbar^2$; but this is compensated exactly by the trace of the energy momentum tensor of the ground state. It is our opinion that this is the property of the cosmological constant at all, also in general relativity.

Furthermore because in (21) the mass M is that of the Higgs-particle, the range l of the potential φ should be very short, so that until now no experimental evidence for the Higgs-gravity may exist, at least in the macroscopic limit. For this reason it also appears unprobable, that it has to do something with the so called fifth force [6]. Finally the factor γ in (22) can be estimated as follows: Taking into consideration the unified theory of electroweak interaction the value of v (see (19a)) is correlated with the mass M_W of the W -bosons according to $v^{-2} = \pi g_2^2 \hbar / M_W^2$ ($g_2 =$ gauge-coupling constant of the group $SU(2)$). Combination with (22) results in

$$(34) \quad \gamma = \frac{g_2^2}{2} \left(\frac{M_P}{M_W} \right)^2 = 2 \times 10^{32}$$

(M_P Planck-mass). Consequently the Higgs-gravity represents a relatively strong scalar gravitational interaction between massive elementary particles,

however with extremely short range and with the essential property of quantizability. If any Higgs-field exists in nature, this gravity is present.

The expression (34) shows that in the case of a symmetry breaking where the bosonic mass is of the order of the Planck-mass, the Higgs-gravity approaches the Newtonian gravity, if the mass of the Higgs-particle is sufficiently small. In this connection the question arises, following Einstein's idea of relativity of inertia, if it is possible to construct a tensorial quantum theory of gravity with the use of the Higgs-mechanism, leading at last to Einstein's gravitational theory in the classical macroscopic limit.

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